

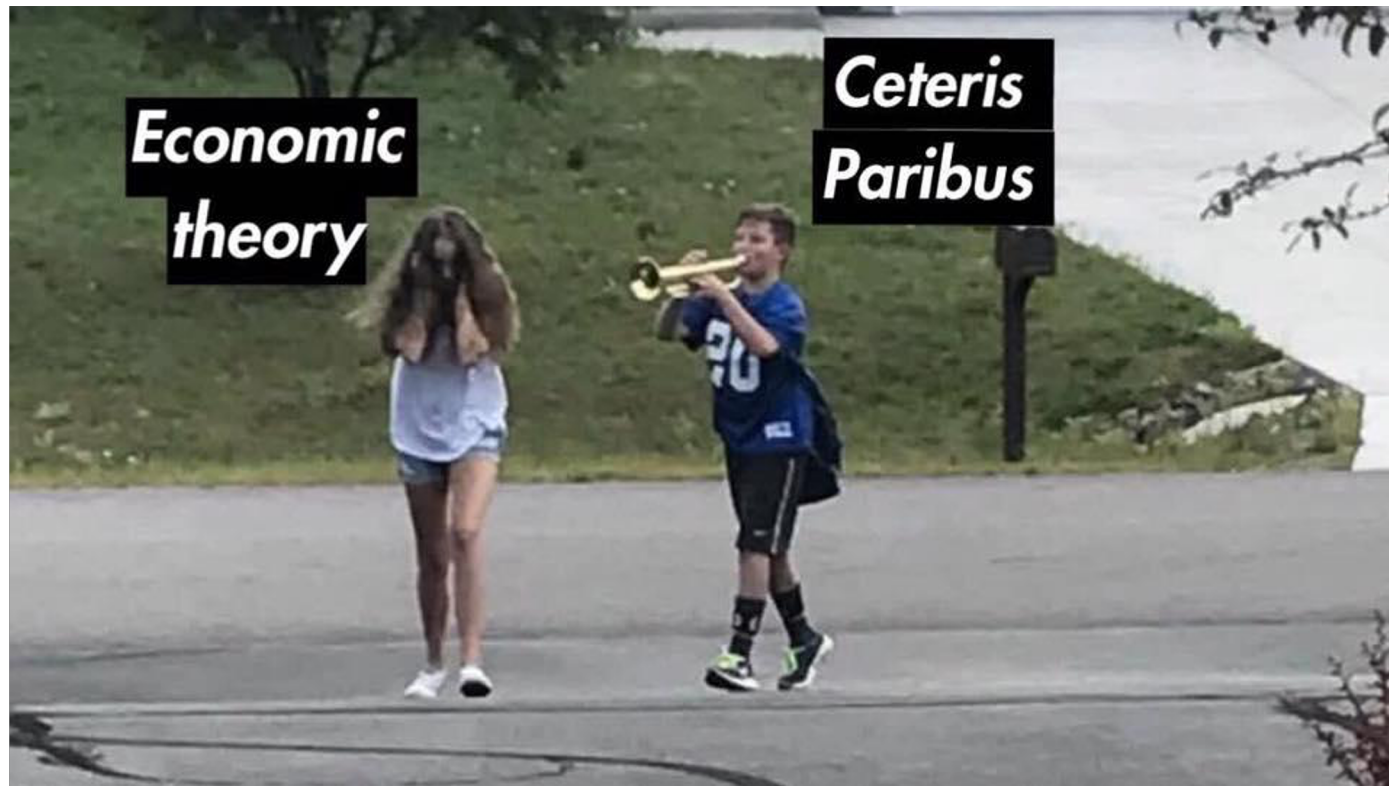
Data 88: Economic Models

Lecture 4: Production

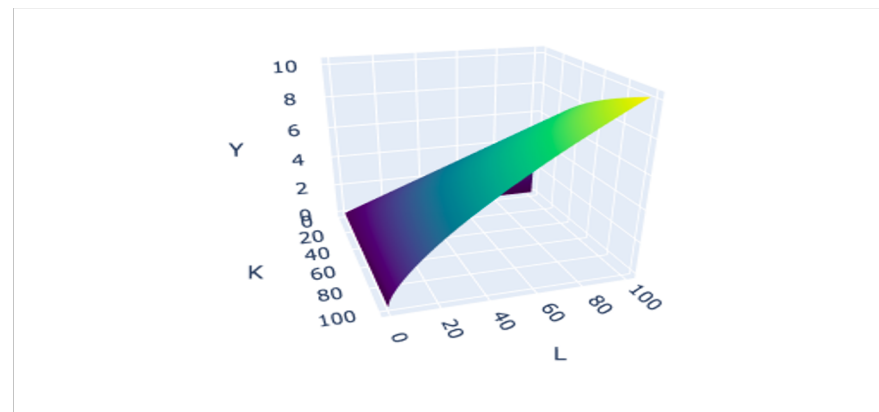
- Project 2 will be released today
- It will be coding-heavy (Data 8 Project 1-esque), but Lab 4 will greatly help you start
- Whole project due (all 3 parts) 2 weeks from now: October 5 @ 11:59pm
 - Office hours are there for you to get help!
 - We will schedule additional office hours if there is demand
 - Recommended: Finish first 2 parts by Sunday, last part next week



- Graduated in May 2020 with degrees in Economics and Data Science
- Taught this course (Data 88), Data 8 and Econ 100B
- Data Scientist with a Singapore-based company
- If you have any questions about Data Science in industry, please feel free to email me at manikui@berkeley.edu



- Overall
 - Factors of Production
 - Total Factor Productivity
 - Cobb-Douglas Production Functions (3D SURFACES!!!!!!)
 - Cross-Country Analysis (this is what the project is)
- Derivation
 - Cobb Douglas
 - Log of Equation
 - Solve for one variable
- Intuition
 - Shifts in A (SLIDERS!!!!!!!!!!)
 - What does α mean?
 - Returns to scale
- Lab time



Name all of the macroeconomic indicators you know

~~★~~ GDP (/capita)

U

π (inflation), CPI

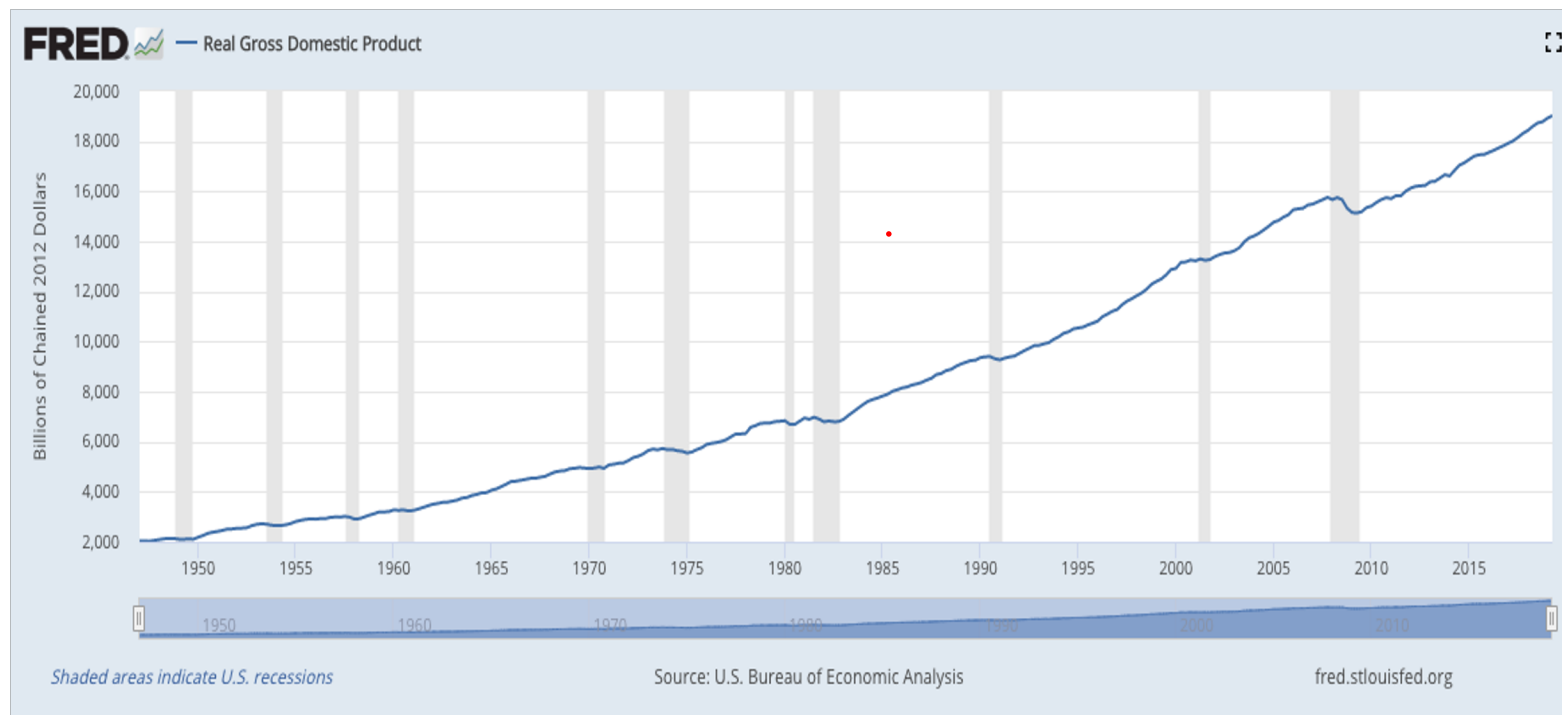
r

IM/EX

CO₂

m1, m2

What is GDP?



How do we calculate it?

- Income approach

rent, wages, r , profits

- Expenditure approach $\rightarrow C + I + G + EX - IM$

- Production approach

- What is production?

inputs $\rightarrow$ $\square$ $\rightarrow$ output
(GDP)

- How do we quantify a country's production?

inputs . prices

- Why would it be important to calculate this?

- what sectors need help



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Let's make a list of **all** the resources that go into producing and selling an iPhone:

metals (K)
labour (slave) (L)
silicon (K)
R & D (A)
shipping (K, L)
sales (L)
I.P. (A)

packaging (K, L)
stores (K)
factories (K)
education (A)
K - capital
L - labour
A - TFP (tech.)



Image source: https://store.storeimages.cdn-apple.com/4982/as-images.apple.com/is/refurb-iphoneX-spacegray_AV1?wid=1144&hei=1144&fmt=jpeg&qlt=80&op_usm=0.5,0.5&.v=1548459945536

A nation's output is a function of the amount of the factors of production that are utilized in its economy; that is to say output is a function of labor and capital, scaled by some value A.

$$f(\underline{K}, \underline{L})$$

$$\cdot \uparrow$$

TFP

$$\underline{Y} = \underline{A} \cdot f(K, L)$$

What could A be? Why is it on the outside of the production function?

- Technology or research and development
- A country with a high TFP can produce far more goods and services than another country with a lower TFP but the same amount of capital and labor

$$A=2 \quad \text{vs.} \quad A=1$$

K, L

- Differences between TFP and other factors of production
 - “Scales” production by some factor A - creates proportional increase in output
 - Technology is non-rivalrous (more than one person can use it)
 - Technology is non-excludable (cannot prevent other people from using it)
 - Ordinal measure
↳ comparison

How the equation was formulated

weighted avg.

Based on the graphs to the right, what would $F(K, L)$ look like?

What assumptions can we make about Y , K and L ?

$$Y = F(K, L) \cdot A$$

$$\ln(Y) = \underline{\text{int}} + \underline{\alpha} \cdot \ln(K) + \underline{\beta} \cdot \ln(L)$$

(Cobb and Douglas did these calculations without computers 😬)

$$Y = \underline{A} \cdot \underline{e}^{\text{int}} \cdot (K^{\alpha} \cdot L^{\beta})$$

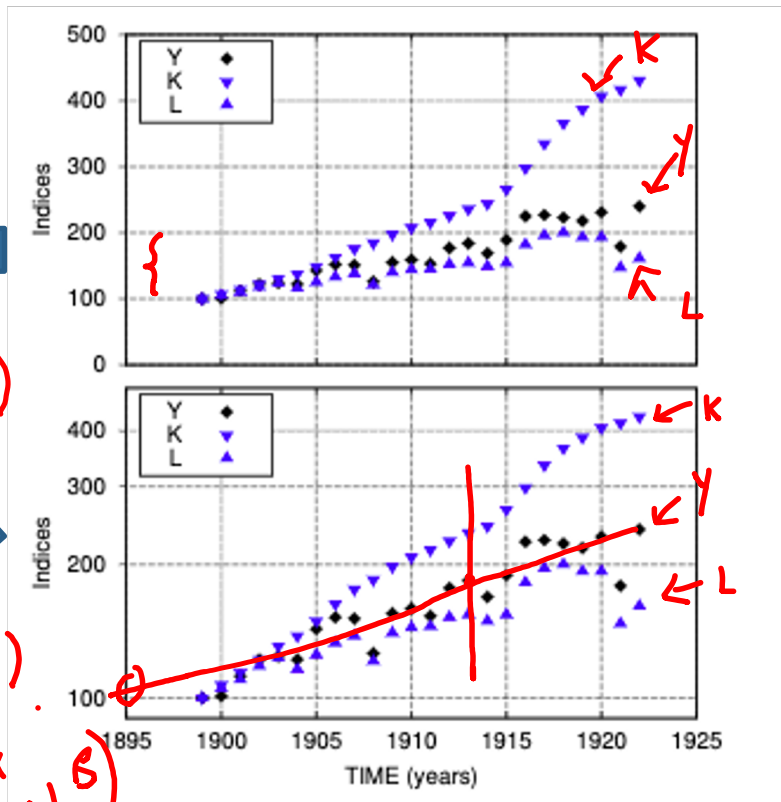
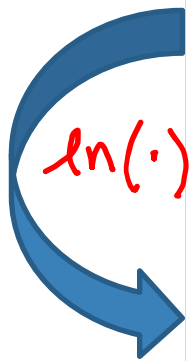


Image source: Hawkins' 100B; Lecture 4, Slide 9

$$f(K, L) = K^\alpha L^\beta$$
$$\Rightarrow \underline{Y} = A \cdot f(K, L) = \underline{A} \underline{K^\alpha L^\beta}$$

- For simplicity sake, we will assume constant returns to scale. That is, $\alpha + \beta = 1$. This allows us to rewrite the function in the following way:

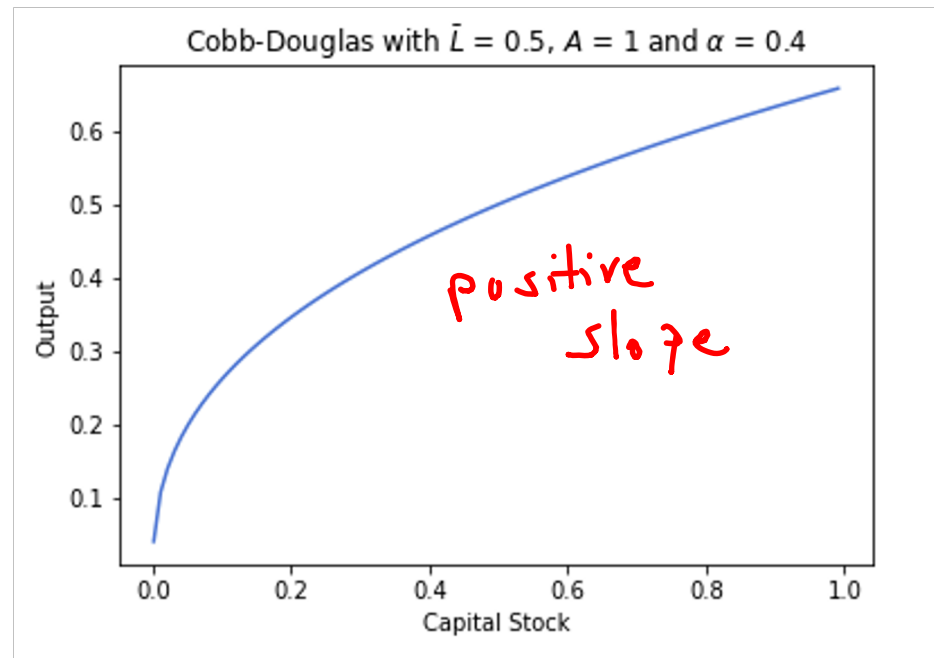
- Where else will we see this function?

← utility

$$Y = AK^\alpha L^{1-\alpha}$$

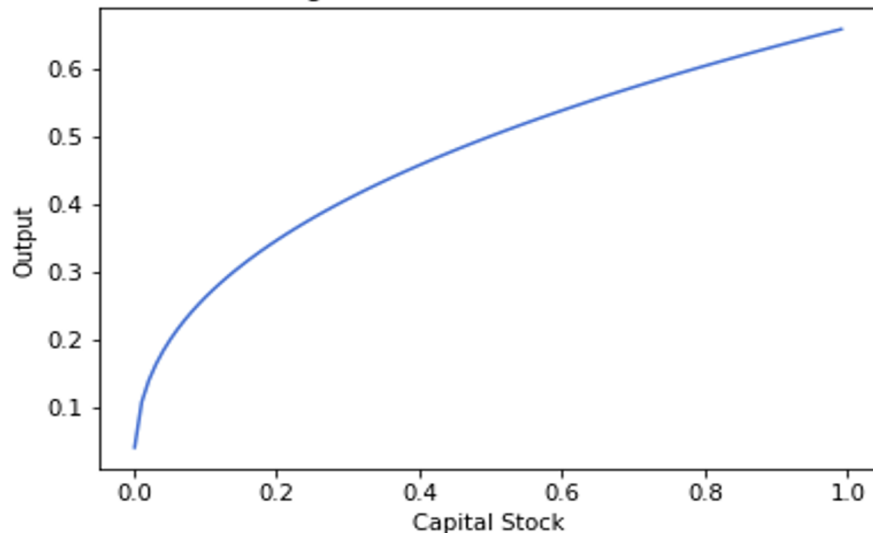
A monetary value of the stock or value of all productive, physical assets in an economy (**not bonds, stocks or other financial products**)

Increasing Returns to Capital
(What does that mean?)



$$Y = AK^\alpha L^{1-\alpha}$$

Cobb-Douglas with $\bar{L} = 0.5$, $A = 1$ and $\alpha = 0.4$



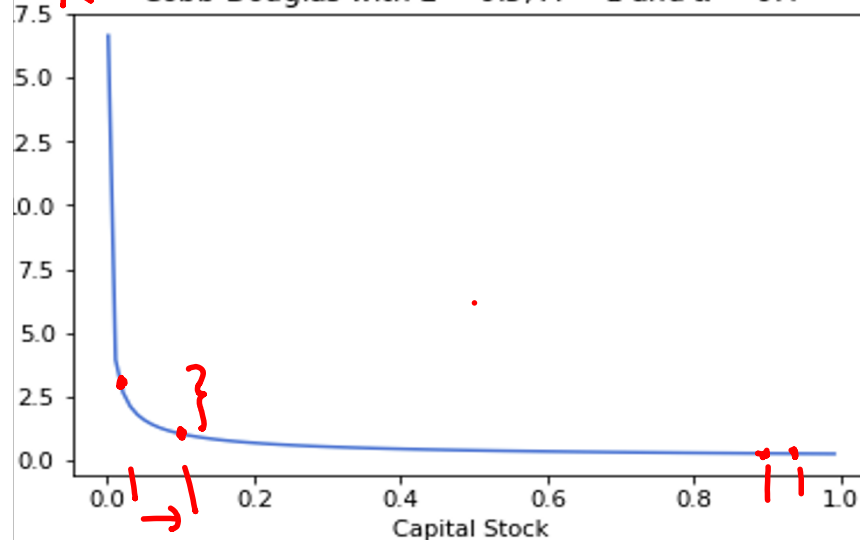
Diminishing marginal returns to capital

What is the intuition behind this?

How do we get the price of capital?

MPK

Cobb-Douglas with $\bar{L} = 0.5$, $A = 1$ and $\alpha = 0.4$



$$MPK = \frac{\partial Y}{\partial K} = A \alpha K^{\alpha-1} L^{1-\alpha}$$

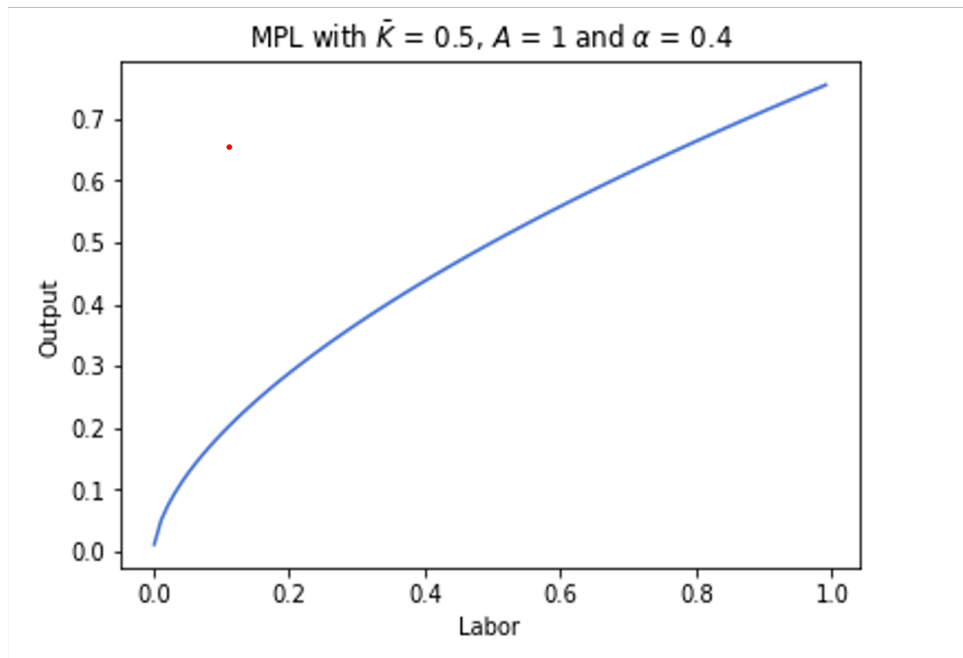
$$= A \alpha \left(\frac{L}{K} \right)^{1-\alpha}$$

$$MR \geq MC$$

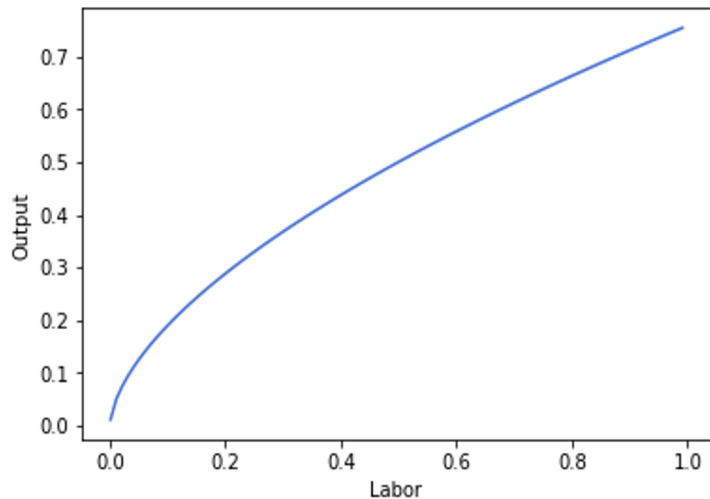
$$MR = MC \Rightarrow \underline{P \cdot MPK = r}$$

The number of hours worked by all individuals who are willing and able to work within a country for a given year

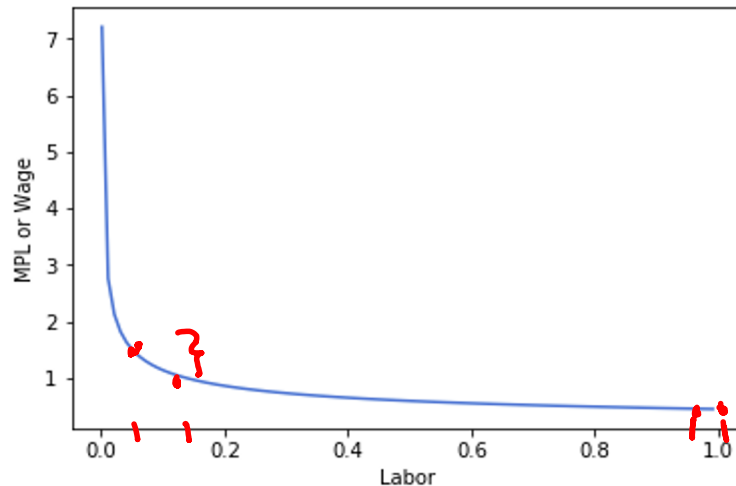
Increasing Returns to Labor
(What does that mean?)



MPL with $\bar{K} = 0.5$, $A = 1$ and $\alpha = 0.4$



MPL with $\bar{K} = 0.5$, $A = 1$ and $\alpha = 0.4$



★ Diminishing Marginal Returns to Labor

What is the intuition behind this?

How do we get the price of labor?

$$MPL = \frac{\partial Y}{\partial L} = A(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}$$

$$MR = MC$$

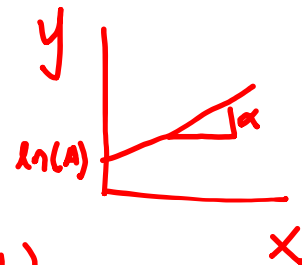
$$P \cdot MPL = W$$

$$Y = A \bar{K}^{\alpha} \bar{L}^B$$

Cobb and Douglas found that a country's output can be modeled very well as a weighted average of the log of capital and labor.

How can we rearrange the function to only be in terms of one variable?

$$Y = A \bar{K}^{\alpha} \bar{L}^{1-\alpha}$$



$$\begin{aligned} \ln(Y) &= \ln(A) + \alpha \ln(K) + (1-\alpha) \ln(L) \\ &= \ln(A) + \alpha \ln(K) + \ln(L) - \alpha \ln(L) \end{aligned}$$

$$\begin{aligned} \ln(Y) - \ln(L) &= \ln(A) + \alpha [\ln(K) - \ln(L)] \\ \ln\left(\frac{Y}{L}\right) &= \ln(A) + \alpha \ln\left(\frac{K}{L}\right) \end{aligned}$$

$$\ln\left(\frac{Y}{L}\right) = \ln(A) + \alpha \ln\left(\frac{K}{L}\right)$$

$$Y = A K^{\alpha} L^{\beta}$$

0.7 0.3

How can we use the above function to compare countries?

① α changes $\rightarrow \beta$ changes $\rightarrow$ Capital vs. labor intensive

② A differences $\rightarrow$ technology

What are possible problems with the function?

α, A fixed

$$Y = A K^{\alpha} L^{\beta}$$

Romer $\Rightarrow \Delta A$ over time

- Supply or Technology shocks
 - Natural environment ← COVID
 - Energy prices
 - Significant financial crises
- What would happen to output?

$\Delta A?$

war

→ gov't intervention

- Can anyone think of any real-world examples?

— OPEC oil crises
— subsidizing technology

What does alpha mean?

Output elasticities of capital and labor: measure the responsiveness of output to a change in the levels of either labor or capital, holding all else constant.

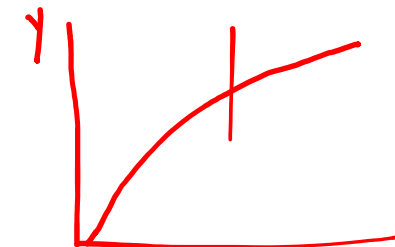
$$Y = AK^{\alpha}L^{1-\alpha}$$

Let us assume constant returns to scale. What does $\alpha > 0.5$ mean?

labour
intensive

Constant Returns to Scale: $\alpha + \beta = 1$

$$c \cdot Y = f(cK, cL) \quad (\star)$$



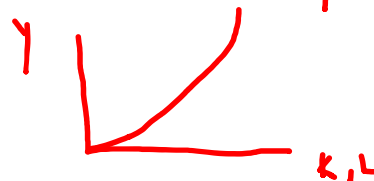
Decreasing Returns to Scale: $\alpha + \beta < 1$

$$cY < f(cK, cL)$$



Increasing Returns to Scale: $\alpha + \beta > 1$

$$cY > f(cK, cL) \quad (\star)$$



What would the last two look like if their production functions were plotted?

Lab Time!

Open up Lab 4 and work through the notebook. We'll be here to help.

The questions will be **very** helpful in preparing your for Project 2!