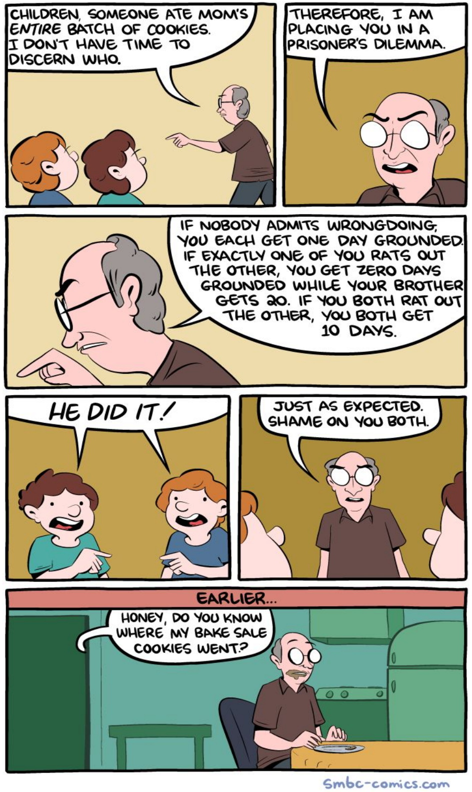
Data 88E: Economic Models

Lecture 8: Game Theory

Announcements

- Project 3 will be released tonight; due Mon 3/22 at 11:59 PM
  - more on this at the end of lecture
- Lab 7 will be released later today; due Mon 3/15 at 11:59 PM


CHILDREN, SOMEONE ATE MOM'S ENTIRE BATCH OF COOKIES. I DON'T HAVE TIME TO DISCERN WHO.
THEREFORE, I AM PLACING YOU IN A PRISONER'S DILEMMA.
IF NOBODY ADMITS WRONGDOING, YOU EACH GET ONE DAY GROUNDED. IF EXACTLY ONE OF YOU RATS OUT THE OTHER, YOU GET ZERO DAYS GROUNDED WHILE YOUR BROTHER GETS 20. IF YOU BOTH RAT OUT THE OTHER, YOU BOTH GET 10 DAYS.
HE DID IT!
JUST AS EXPECTED. SHAME ON YOU BOTH.
EARLIER...
HONEY, DO YOU KNOW WHERE MY BAKE SALE COOKIES WENT?
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Lecture 7 Outline

- Game Theory Overview
- Strategy
- Prisoner's Dilemma
- Nash Equilibria
- Oligopolies
- Cournot Competition
- Bertrand Competition
- Iterated Prisoner's Dilemma
- A Healthy Dose of Irony: Webcam Placement Study

Game Theory

- Branch of mathematics concerned with the strategic interaction of rational decision-makers
- Different types of games:
    - cooperative vs. non-cooperative
    - symmetric vs. asymmetric
    - simultaneous vs. sequential
    - perfect vs. imperfect information
- We will study simultaneous games of perfect information.

Strategies

- Systematic methods of playing games
  - tell players what move to make based on available information
  - essentially a probability distribution over a player's choices
- Different types
  - **pure** strategies put all probability on a single choice
  - **mixed** strategies divide probabilities over an array of choices
- Strategies can be random but often aren't
  - expected utility is a common counter-strategy for randomness
  - Nash equilibria (if they exist) tell us the "best" move to make in games of imperfect information

Aside: Payoff Matrices

Alan is trying to schedule his classes and wants to enroll in two classes with conflicting final exam times. One class will not give accommodations and is required for his degree. Alan knows the other class will give an accommodation with probability 0.6. Consider the payoff matrix below.

		Other Class	
		Give accommodations	Don’t give accommodations
Alan	Enroll in both classes	(10, 10)	(-10, 8)
	Don’t enroll in both classes	(2, 5)	(5, 0)

Expected Utility

		Other Class	
		Give accommodations	Don't give accommodations
Alan	Enroll in both classes	(10, 10)	(-10, 8)
	Don't enroll in both classes	(2, 5)	(5, 0)

Expected Utility Hypothesis

- The **expected utility** of a set of *n* outcomes x_i is the average utility of each outcome $u(x_i)$ weighted by the probability of that outcome's occurrence p_i:

$$E[u(x)] = \sum_{i=1}^{n} p_i u(x_i)$$

- The **expected utility hypothesis** says that when an individual makes a gamble, they will choose the option that maximizes their expected utility
  - the utility of each option is based on their preferences
- Expected utility maximization (under the EUH) is one way of strategizing in the face of randomness

Prisoner's Dilemma

- Prolific game first considered in the 1950s
- Single-round symmetric simultaneous non-cooperative game of perfect information
- Each prisoner can either cooperate (maintain their silence) or defect (talk to the police)
    - Does this sound familiar? You experienced a multiplayer PD on test 1!
- *Reading the payoff matrix:* a higher value (more years in prison) is worse! our goal is to minimize the values in the payoff matrix

		Prisoner B	
		Cooperate	Defect
Prisoner A	Cooperate	(2, 2)	(5, 0)
	Defect	(0, 5)	(4, 4)

Nash Equilibrium

A set of strategy choices in a **non-cooperative** game in which no player can increase their payoff by unilaterally changing their strategy.

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What about the prisoner's dilemma?

		Prisoner B	
		Cooperate	Defect
Prisoner A	Cooperate	(2, 2) 1	(5, 0) 2
	Defect	(0, 5) 3	(4, 4) 4

1) C, C: not a NE either PA or PB can switch to get a better payoff

2) C, D: not a NE b/c PA can change to get 4 years

3) D, C: not a NE: same (2) but for PB

4) D, D: Yes.

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(best payoff)

minimum for PD

Nash equilibria can be read from payoff matrices: to see if a game state is a Nash equilibrium, the first value in the cell must be the ~~maximum~~ among all first values in that column and the second value must be the ~~maximum~~ across all second values in that row.

Oligopolies

- Markets in which the number of participants is limited, e.g. airlines
- Allow their participants to function collectively in a manner similar to a monopoly
  - when this collective monopoly is illegal, these are referred to as **cartels**

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Oligopolies

- Within oligopolies, however, we can observe competition
  - participants try to take market share by lowering prices below the agreed-upon floor
  - e.g. 2020 oil price war that was started when Saudi Arabia lowered its prices below the OPEC price in response to Russia refusing to reduce production

Cournot Competition

- Model of a market in which firms compete by changing their output
    - market has a fixed number of firms that produce the same product
    - firms do not collude but have market power
    - each firm knows the number of firms and has its own cost function used to determine its level of output
    - firms share a single, fixed marginal cost

duopolies

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- OPEC is a ~~good~~ example of a Cournot oligopoly
  - its members affect market prices by changing their level of production
  - but this shows a flaw: Cournot equilibrium implies that collusion is the rational policy, but game theory shows us that ~~cartel~~ members undercut one-another

Cournot Competition

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$P(Q)$ = market demand $\quad q_1$ = firm 1 quantity $\quad q_2$ = firm 2 quantity

$Q = q_1 + q_2$

Eq'm price that both firms sell at $P_1 = P_2 = P(q_1 + q_2)$

how to find q_1^*, q_2^*?

Let's say firm 1 believes firm 2 will produce $q_2 = 9$

draw firm 1's residual demand curve:

$q_2 = 10$

$d_1 = P(q_1 + q_2)$

$d_1 = P(q_1 + 10) = m(q_1 + 10) + b = mq_1 + 10m + b$

Cournot Competition

q_1^* where firm 1's marginal revenue intersects c

$P(Q) = mQ + b$

$MR = \frac{d}{dQ} TR = \frac{d}{dQ} P(Q) \cdot Q$

$= \frac{d}{dQ} (mQ^2 + bQ)$

$r = 2mQ + b$

$d_1(q_1) = P(q_1 + q_2)$

$= mq_1 + mq_2 + b$

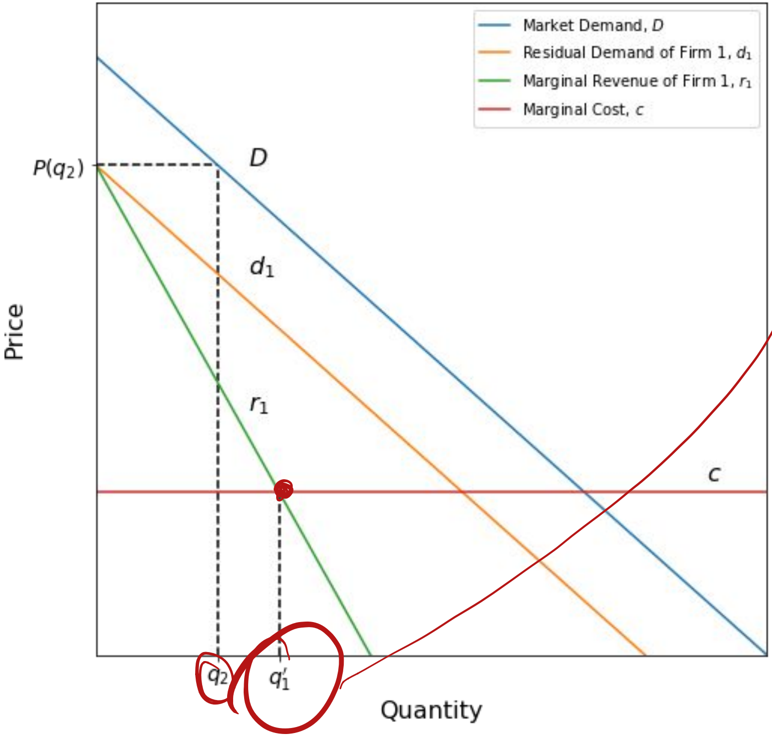
$r_1 = 2mq_1 + mq_2 + b$

$= 2 d_1(q_1) - d_1(0)$

$2(mq_1 + mq_2 + b)$

$-(mq_2 + b)$

Cournot Competition



optimal output level of firm 1 assuming firm 2 produces at q_2

Cournot Competition

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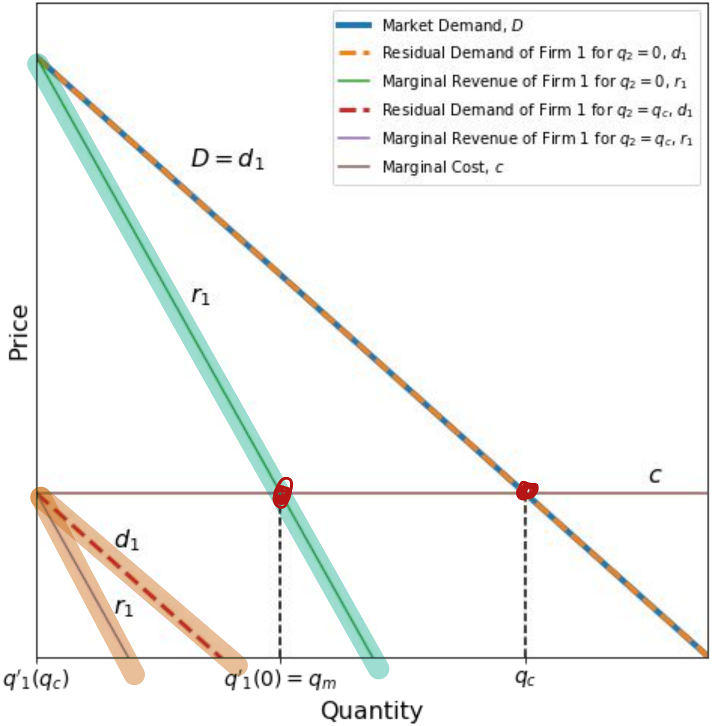
We've found q_1 at a fixed q_2.

What is the eq'm?

Find this by defining $q_1(q_2)$ that tells us the quantity firm 1 produces at for a given q_2.

This is a line, so we need 2 points.

Cournot Competition



q_c = perfect competition quantity

q_m = monopoly quantity

when $q_2 = 0$, $q_1'(0) = q_m$

when $q_2 = q_c$, $q_1'(q_c) = 0$

Cournot Competition

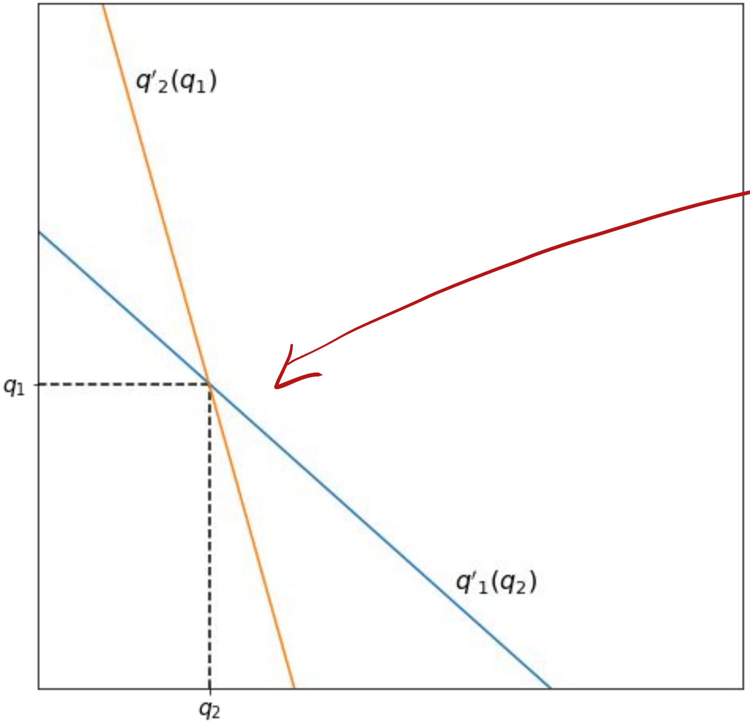
$$q_1'(0) = q_m \qquad q_1'(q_c) = 0$$

$$q_1'(q_2) = \frac{-q_m}{q_c} q_2 + q_m$$

What about $q_2'(q_1)$? inverse of q_1'!

$$q_2'(q_1) = \frac{-q_c}{q_m} q_1 + q_c$$

Cournot Competition



(q_2^*, q_1^*)

Cournot Competition

Bertrand Competition

- Similar to Cournot, but instead firms compete by changing price rather than output
  - assumes consumers want to pay the lowest price for goods
  - they will buy everything at the lower price
    - assumes that the firm offering the lower price has the capacity to meet the demand of the entire market at that price
    - if prices are equal, demand will be split evenly among them
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- Coke vs. Pepsi is a good example of a Bertrand duopoly
  - both firms compete on price and take into account the price of their competitor

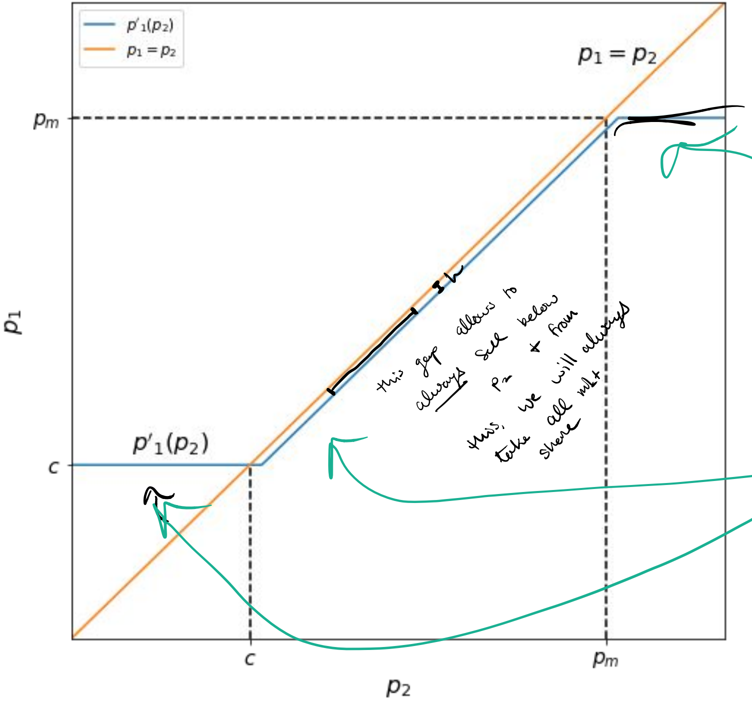
Bertrand Competition

P_m = monopoly price level

$MR = 2P(Q) - P(Q) = c$

how to sell at a profit <u>but</u> below P_2?

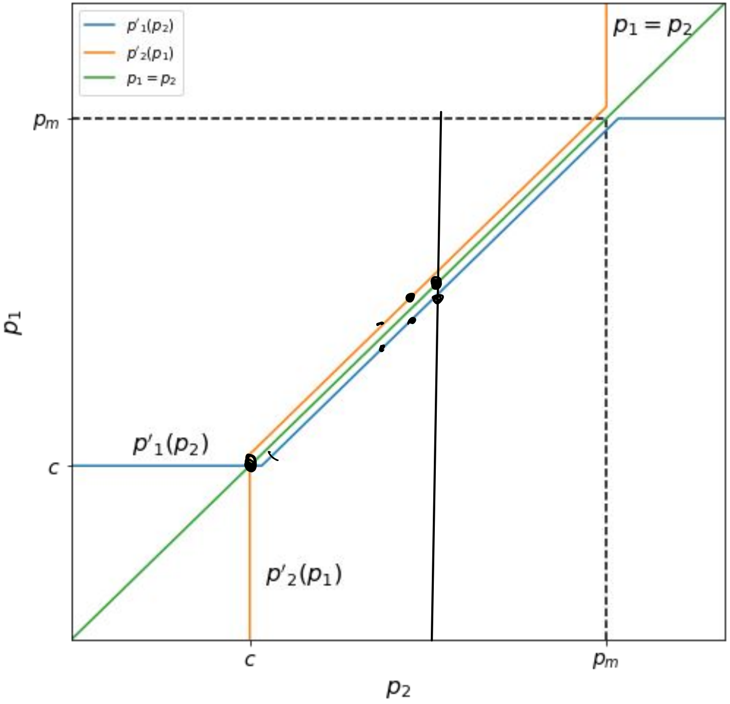
Bertrand Competition



Solution: introduce a small positive difference

$$p_1'(p_2) = \begin{cases} c & \text{if } p_2 < c+h \\ p_2 - h & \text{if } c+h \le p_2 < p_m + h \\ p_m & \text{if } p_2 \ge p_m + h \end{cases}$$

Bertrand Competition



Where is the eq'm?

$$p_1^* = p_2^* = c$$

Bertrand Competition

- Implications:
  - even a duopoly in a market is enough competition to push prices down to the level of perfect competition
  - but this model relies on serious assumptions
    - there are many reasons why consumers might not buy the lowest-priced item (e.g. non-price competition, search costs)
    - ignores the fact that firms may not be able to supply the entire market
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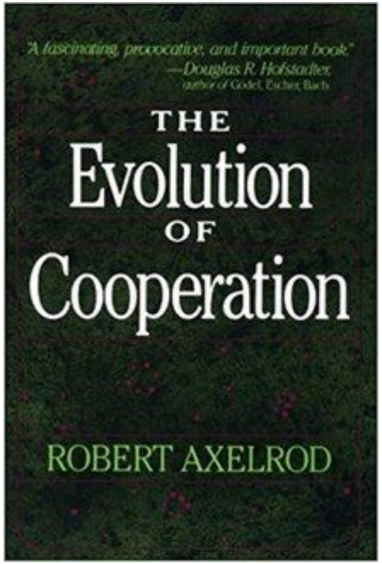
  - the model also demonstrates big incentives to cooperate and raise prices to the monopoly level
    - but this is not the Nash equilibrium of the model

↳ PD again

Iterated Prisoner’s Dilemma

- Paradigm in which the prisoner’s dilemma is played repeatedly over multiple rounds
- Players know the history of their opponent but not their current move
- Was used to model the Cold War and MAD in the 1950’s
  - different strategies for playing this game were analyzed in order to see which was most effective at winning the game
- A famous tournament for the Iterated Prisoner’s Dilemma was run by Robert Axelrod
  - the winning strategy at this tournament was tit-for-tat
  - excellent episode of [Radiolab] on this tournament



Project 3

In Project 3, you will

- study the iterated prisoner's dilemma and recreate Axelrod's tournament
- study an experiment by Thomas and Pemstein (2015) relating the perception of height to game strategies
- recreate the analysis of Thomas and Pemstein (2015) using A/B testing
    - webcam placement influences leader-follower behavior in games